Written Exam at the Department of Economics winter 2020-21 R

Contract Theory

22 February 2021

(4 hour open book exam)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

This exam question consists of 5 pages in total

This exam has been changed from a written Peter Bangsvej exam to a take-home exam with helping aids. Please read the following text carefully in order to avoid exam cheating.

Be careful not to cheat at exams!
You cheat at an exam, if you during the exam:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text. This also applies to text from old grading instructions.
- Make your exam answers available for other students to use during the exam
- Communicate with or otherwise receive help from other people
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Use parts of a paper/exam answer that you have submitted before and received a passed grade for without making use of source referencing (self plagiarism)

You can read more about the rules on exam cheating on the study information pages in KUnet and in the common part of the curriculum section 4.12.

Exam cheating is always sanctioned with a warning and dispelling from the exam. In most cases, the student is also expelled from the university for one semester.

Attempt both questions.

Explain all the steps of your analysis and define any new notation that you use.

Show all the calculations that your analysis relies on.

Question 1: Model hazard with additional information about the agent's behavior

The following is an extension of the 2x2 moral hazard model with a risk neutral agent (protected by limited liability) that we studied in the course. In the extension, the principal has access to an additional report indicating whether or not the agent has chosen a high effort.

Consider a moral hazard model where the principal and the agent are both risk neutral but where the agent is protected by limited liability. The principal is in charge of a project, although he is unable to take care of all the practical things related to the project himself. Instead, the principal delegates the task of running the project to an agent. The project gives rise to either a large surplus ($S = \overline{S}$) or a small surplus ($S = \underline{S}$, with $\overline{S} > \underline{S} > 0$). The likelihood of each outcome depends on whether the agent chooses a low effort ($e = 0$) at no cost, or a high effort ($e = 1$) at cost $\psi > 0$. Specifically, given an effort level $e \in \{0, 1\}$, the probability that a large surplus is realized equals π_e. (Accordingly, the probability of a small surplus is $1 - \pi_e$.) Choosing a high effort increases the likelihood of a large surplus, and both probabilities take values strictly between zero and one: $0 < \pi_0 < \pi_1 < 1$.

The principal cannot observe the agent's effort choice directly or perfectly. However, the principal receives a report r that indicates either a 0 (as in "low effort") or an N (as in "no information"). If $e = 1$, the report shows $r = N$ with probability one. If $e = 0$, the report shows $r = 0$ with probability $\gamma \in (0, 1)$. The principal can also observe the surplus that has realized.

The principal can commit to making a monetary payment t to the agent that is contingent on the surplus level and the content of the report. This means that the payment can be contingent on four different events:

- $S = \underline{S}$ and $r = 0$ (denote the payment after this event by t_1);
- $S = \overline{S}$ and $r = 0$ (denote the payment after this event by t_2);
- $S = \underline{S}$ and $r = N$ (denote the payment after this event by t_3);
- $S = \overline{S}$ and $r = N$ (denote the payment after this event by t_4).

Both the principal and the agent are risk neutral, but the agent is protected by limited liability: $t_1 \geq 0$, $t_2 \geq 0$, $t_3 \geq 0$, and $t_4 \geq 0$. The agent's outside option yields a payoff of zero. The timing of events is as follows. (i) The principal chooses t_1, t_2, t_3, and t_4, trying to maximize the expected surplus that is generated, net of the expected payments to the agent. (ii) The agent decides whether or not to accept the contract offer. Her objective is to maximize her expected payment t, minus the effort cost (if any). (iii) If rejecting the contract offer, the agent receives her outside option payoff (and the principal receives the same zero payoff). If the agent accepts the offer, she chooses the effort level.

Suppose the principal wants to induce the outcome $e = 1$. Answer the following questions.

(a) Write down expressions for the objective function and the constraints in the principal's optimization problem. One of the constraints should be an individual rationality constraint, which we refer to as the "IR constraint".

(b) Show that the IR constraint is implied by other constraints in the problem.

(c) What are the optimal choices of t_1, t_2, t_3, and t_4?

Question 2: Private information about both an exogenous effort cost and an endogenous effort choice

The following is a hybrid model of adverse selection and moral hazard, where the agent has private information about both her effort choice and an exogenously given parameter in her effort cost function.

The principal of the model is in charge of a project, although he is unable to take care of all the practical things related to the project himself. Instead, the principal delegates the task of running the project to an agent. The project gives rise to either a large surplus ($S = \overline{S}$) or a small surplus ($S = \underline{S}$, with $\overline{S} > \underline{S} > 0$). The likelihood of each outcome depends on the agent's effort $e \in [0, 1]$. Specifically, $\Pr[S = \overline{S}] = e$. By choosing an effort level e, the agent incurs the effort cost $\psi(e, \theta)$, where θ is a parameter that determines how costly it is (in absolute terms and on the margin) for the agent to choose a particular effort level. We make the following assumptions about the effort cost function:

$$\psi(0, \theta) = 0, \qquad \psi_1(e, \theta) > 0, \qquad \psi_{11}(e, \theta) > 0, \qquad \psi_2(e, \theta) > 0, \qquad \psi_{12}(e, \theta) > 0 \qquad \text{for all } e \in (0, 1].$$

In addition, we assume that the following Inada conditions hold: $\psi_1(0, \theta) = 0$ and $\psi_1(1, \theta) = \infty$. The cost parameter θ can take two values: $\theta \in \{\theta_A, \theta_B\}$, with $0 < \theta_A < \theta_B$. The agent knows the value of θ perfectly. However, the principal only knows that

$$\Pr[\theta = \theta_A] = \nu \qquad \text{and} \qquad \Pr[\theta = \theta_B] = 1 - \nu,$$

with $0 < \nu < 1$. Moreover, the principal cannot observe the agent's effort choice e. However, both the principal and the agent (as well as an outside court) can observe the magnitude of the realized surplus, $S \in \{\underline{S}, \overline{S}\}$.

The principal has all the bargaining power and makes a take-it-or-leave-it offer to the agent. A contract can specify the payment $\underline{t}$ that the agent will receive after a small surplus has realized, and the payment $\bar{t}$ that the agent will receive after a large surplus has realized. The principal is risk neutral and his payoff, given a surplus S and a payment t, equals $V = S - t$. The agent is also risk neutral and her payoff, given a payment t and an effort e, equals $U = t - \psi(e, \theta)$. The agent's outside option (the same for the two types) would yield the payoff zero.

The principal offers a menu of two distinct contracts to the agent. The contract variables are indicated either with a subscript A or B, depending on which agent type the contract is aimed at. Moreover, the contract variables are indicated either with "upper-bars" or "lower-bars", depending on which observed surplus level they are contingent on. Given this notation, we can write the principal's expected payoff as

$$EV = \nu \left[e_A(\overline{S} - \bar{t}_A) + (1 - e_A)(\underline{S} - \underline{t}_A)\right] + (1 - \nu)\left[e_B(\overline{S} - \bar{t}_B) + (1 - e_B)(\underline{S} - \underline{t}_B)\right]. \quad (1)$$

The optimal contract menu maximizes the expected payoff in (1) with respect to the variables

$$\left(\underline{t}_A, \bar{t}_A, \underline{t}_B, \bar{t}_B, e_A, e_A^d, e_B, e_B^d\right),$$

subject to the following constraints:

$$e_A \bar{t}_A + (1 - e_A)\underline{t}_A - \psi(e_A, \theta_A) \geq 0, \quad \text{(IR-A)}$$

$$e_B \bar{t}_B + (1 - e_B)\underline{t}_B - \psi(e_B, \theta_B) \geq 0, \quad \text{(IR-B)}$$

$$e_A \bar{t}_A + (1 - e_A)\underline{t}_A - \psi(e_A, \theta_A) \geq e_A^d \bar{t}_B + (1 - e_A^d)\underline{t}_B - \psi\left(e_A^d, \theta_A\right), \quad \text{(IC-A)}$$

$$e_B \bar{t}_B + (1 - e_B)\underline{t}_B - \psi(e_B, \theta_B) \geq e_B^d \bar{t}_A + (1 - e_B^d)\underline{t}_A - \psi\left(e_B^d, \theta_B\right), \quad \text{(IC-B)}$$

$$\underline{t}_A \geq 0, \qquad \bar{t}_A \geq 0, \qquad \underline{t}_B \geq 0, \qquad \bar{t}_B \geq 0, \quad \text{(LL)}$$

$$\bar{t}_A - \underline{t}_A = \psi_1(e_A, \theta_A), \quad \text{(FOA-A)}$$

$$\bar{t}_B - \underline{t}_B = \psi_1\left(e_A^d, \theta_A\right), \quad \text{(FOA-A-d)}$$

$$\bar{t}_B - \underline{t}_B = \psi_1(e_B, \theta_B), \quad \text{(FOA-B)}$$

$$\bar{t}_A - \underline{t}_A = \psi_1\left(e_B^d, \theta_B\right). \quad \text{(FOA-B-d)}$$

Answer the following questions.

(a) Explain in words what each one of the four last constraints (i.e., the ones with a label starting with "FOA") says and how we can understand the two variables e_A^d and e_B^d.

(b) Show that the IR-A constraint is implied by model assumptions and other constraints in the problem.

One can show that also the IR-B constraint is implied by model assumptions and other constraints in the problem. In addition, suppose that, at the optimum, the LL constraints $\overline{t}_A \geq 0$ and $\overline{t}_B \geq 0$ are lax, while the two remaining LL constraints are both binding: $\underline{t}_A = \underline{t}_B = 0$. The problem can now be written as follows. Maximize

$$EV = \nu\left[e_A\overline{S}_A + (1-e_A)\underline{S} - e_A\psi_1\left(e_A,\theta_A\right)\right] + (1-\nu)\left[e_B\overline{S}_B + (1-e_B)\underline{S} - e_B\psi_1\left(e_B,\theta_B\right)\right]$$

with respect to e_A and e_B, subject to the following two constraints:

$$e_A\psi_1\left(e_A,\theta_A\right) - \psi\left(e_A,\theta_A\right) \geq e_A^d\psi_1\left(e_B,\theta_B\right) - \psi\left(e_A^d,\theta_A\right), \tag{IC-A}$$

$$e_B\psi_1\left(e_B,\theta_B\right) - \psi\left(e_B,\theta_B\right) \geq e_B^d\psi_1\left(e_A,\theta_A\right) - \psi\left(e_B^d,\theta_B\right). \tag{IC-B}$$

Moreover, the variable e_B^d is implicitly defined, as a function of e_A, by the identity

$$\psi_1\left(e_A,\theta_A\right) \equiv \psi_1\left(e_B^d,\theta_B\right). \tag{2}$$

Similarly, the variable e_A^d is implicitly defined, as a function of e_B, by the identity

$$\psi_1\left(e_B,\theta_B\right) \equiv \psi_1\left(e_A^d,\theta_A\right). \tag{3}$$

The Lagrangian associated with the above problem can be written as

$$\begin{aligned}\mathcal{L} = {} & \nu\left[e_A\overline{S}_A + (1-e_A)\underline{S} - e_A\psi_1\left(e_A,\theta_A\right)\right] + (1-\nu)\left[e_B\overline{S}_B + (1-e_B)\underline{S} - e_B\psi_1\left(e_B,\theta_B\right)\right] \\ & + \lambda_A\left[e_A\psi_1\left(e_A,\theta_A\right) - \psi\left(e_A,\theta_A\right) - e_A^d\psi_1\left(e_B,\theta_B\right) + \psi\left(e_A^d,\theta_A\right)\right] \\ & + \lambda_B\left[e_B\psi_1\left(e_B,\theta_B\right) - \psi\left(e_B,\theta_B\right) - e_B^d\psi_1\left(e_A,\theta_A\right) + \psi\left(e_B^d,\theta_B\right)\right],\end{aligned}$$

where $\lambda_A \geq 0$ and $\lambda_B \geq 0$ are the Lagrange multipliers associated with IC-A and IC-B, respectively, and e_B^d and e_A^d are implicitly defined by (2) and (3), respectively.

(c) Denote the optimal value of e_A by e_A^{SB}. Solve as much as you need of the problem in order to argue that, if λ_A is small enough, then $e_A^{SB} < e_A^{FB}$, where e_A^{FB} is the A type's first-best effort level (i.e., e_A^{FB} is implicitly defined by $\overline{S} - \underline{S} = \psi_1\left(e_A^{FB},\theta_A\right)$). Assume that all second-order conditions associated with the problem are satisfied.

End of Exam